Appendix C – Establishing prices in the Frequency Assignment Stage (mathematical description)

This document is a non-binding translation to English of the Swedish appendix (to the Open Invitation Part 2) published 28 May 2025.

In order to determine the prices in the Frequency Assignment Stage (section 4.3.4.6), the following optimisation problem will be solved.

Notation:

W	Set of bidders who have won blocks and from whom bids are collected in the Frequency Assignment Stage.
i	An index for all bidders in W.
C	A subset of the bidders in W (i.e., $C \subseteq W$).
β_i^*	Amount of the winning assignment bid from bidder i, i.e., the amount bid for the assignment option that corresponds to the assignment of the bidder in the winning assignment.
v^{-C}	Maximum value of the assignment bids that would be associated with the hypothetical winning assignment that would be selected if all bidders in C were deemed to have made a bid of zero for all their assignment options. Note that $v^{-W} = 0$, i.e., if all bidders were deemed to have made zero assignment bids, the total value of the bids associated with the winning permissible assignment (which would be determined at random) would be zero. Note that $v^{-\emptyset} = \sum_{i \in W} \beta_i^*$, where $\emptyset$ is the empty set.

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$\sigma(C)$	Opportunity cost of assigning the bidders in $C \subseteq W$ the assignment options they obtain in the winning assignment. This is calculated as: $$\sigma(C) = v^{-C} - \sum_{i \in W \setminus C} \beta_i^*$$ Note that, with $C = \{i\}$, i.e., the set containing a single bidder i, $\sigma(\{i\})$ is the individual opportunity cost of this bidder receiving its winning assignment. This is referred to as the bidder's "individual opportunity cost".
p_i	The price to be paid by bidder i in the winning assignment.

Optimization Problem:

Step 1: Find a vector p^* of prices that minimizes the total payments for the selected assignment options by solving the following optimization problem:

$$\min \sum_{i \in W} p_i \quad \text{subject to:} \quad \sum_{i \in C} p_i \geq \sigma(C) \ \ \forall\, C \subseteq W.$$

Step 2: If $\sum_{i \in W} p_i^* = \sum_{i \in W} \sigma(\{i\})$ this solution is unique and each bidder will pay its individual opportunity cost, i.e., $p_i^* = \sigma(\{i\}) \ \forall\, i$.

Since each bidder i must pay at least its individual opportunity cost $\sigma(\{i\})$ according to the constraint in step 1 where $C = \{i\} \subseteq W$, this equality can only hold if each bidder i pays exactly its individual opportunity cost, $p_i^* = \sigma(\{i\})$.

Step 3: Otherwise, i.e., if $\sum_{i \in W} p_i^* > \sum_{i \in W} \sigma(\{i\})$, the individual assignment prices are determined by solving the following minimization problem:

$$\min \sum_{i \in W} \left(p_i - \sigma(\{i\})\right)^2 \quad \text{subject to:} \quad \sum_{i \in C} p_i \geq \sigma(C) \ \ \forall\, C \subseteq W, \quad \sum_{i \in W} p_i = \sum_{i \in W} p_i^*.$$

This quadratic program has a unique solution: the minimum price vector defining the point in Euclidean space, with one dimension for each bidder's assignment price, that satisfies the constraints in step 3 and is closest to the point defined by the vector of individual opportunity costs.

Step 4: Round up p_i to the nearest full SEK amount.