Appendix E – Establishing prices in the placement stage (mathematical description)

This document is a non-binding translation to English of the Swedish appendix (to the Open Invitation Part 2) published 26 April 2023

The following optimization problem will be used, for each frequency band's placement procedure, to determine prices in the placemen stage (section 4.4.3.6).

Notation:

W	Set of bidders who have won frequencies in the respective band and from whom placement bids are collected.
i	An index for all bidders in W.
C	A subset of the bidders in W (i.e. $C \subseteq W$).
$\beta_i^\star$	Amount of the successful placement bid from bidder i, i.e. the amount bid for the placement option that corresponds to the placement of the bidder in the winning band plan.
v^{-C}	Maximum value of the placement bids that would be associated with the hypothetical winning band plan that would be selected if all bidders in C were deemed to have made a bid of zero for all their placement options (or not have made any placement bids). Note that $v^{-W} = 0$, i.e. if all bidders were deemed to have made zero placement bids the total value of the bids associated with the winning band plan (which would be determined at random) would be zero. Note that $v^{-\emptyset} = \sum_{i \in W} \beta_i^\star$, where $\emptyset$ is the empty set.
$\sigma(C)$:	Opportunity cost of assigning the bidders in $C \subseteq W$ the placement options they obtain in the winning band plan. These are calculated as $\sigma(C) = v^{-C} - \sum_{i \in C} \beta_i^\star$. Note that, with $C = \{i\}$, i.e. an individual bidder i, $\sigma(\{i\})$ is the individual opportunity cost of this bidder receiving its placement in the winning band plan. We refer to this as the bidder's 'individual opportunity cost'.
p_i	Price to be paid by bidder i for its placement in the winning band plan, with the vector of prices for all bidders being denoted as p.

Step 1: Establish the vector $p^\star$ of prices that minimises total payments for the selected placement options by solving the following optimisation problem:

$$\min \sum_{i \in W} p_i$$

subject to:

$$\sum_{i \in C} p_i \geq \sigma(C) \forall C \subseteq W$$

Step 2: If $\sum_{i \in W} p_i^\star = \sum_{i \in W} \sigma(\{i\})$ this solution is unique[1] and each bidder will have to pay its individual opportunity cost, i.e. $p_i^\star = \sigma(\{i\}) \forall i$.

Step 3: Otherwise, i.e. if $\sum_{i \in W} p_i^\star > \sum_{i \in W} \sigma(\{i\})$, determine individual placement prices by solving the following minimisation problem:

$$\min \sum_{i \in W} \left(p_i - \sigma(\{i\})\right)^2$$

subject to:

$$\sum_{i \in C} p_i \geq \sigma(C) \forall C \subseteq W$$

$$\sum_{i \in W} p_i = \sum_{i \in W} p_i^\star$$

This quadratic program has a unique solution (the minimum price vector defining the point in Euclidean space that satisfies the constraints in step 3 and that is closest to the point defined by the vector of individual opportunity costs).

Step 4: Round up each p_i to the nearest full SEK amount.

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[1] Uniqueness follows from the fact that the constraints applied to the minimisation in step 1 include a constraint for each bidder that requires the bidder to pay at least its individual opportunity cost. Therefore, the equality in step 2 can only hold if every bidder pays exactly its individual opportunity cost, which is then the unique solution to the minimisation problem from step 1.